Phil 2310 Assignment 10
Fall 2010

This homework is due by the beginning of class on Wed, Dec 1st. Note that there are two pages to the homework.

Part I: The Theory of Tarski's World Blocks
Chap 10: The Axiomatic Method
Read 10.5: pages 283-288.
Do 10.30

Chap 12: Axiomatizing Shape
Read 12.5: pages 338-341.
Do 12.30-12.38.

For 12.30-12.36 you can use informal, formal, or a mixture of formal and informal techniques. For 12.37, note that a dodecahedron has twelve sides, a cube has six sides, and a tetrahedron has four sides. For 12.38, the six size predicates are: Small, Medium, Large, Smaller, Larger, and SameSize. Small, Medium, and Large can be modeled exactly as Cube, Tet, and Dodec are and SameSize exactly as SameShape. I would model Larger just like I did MoreSides in 12.37 and Smaller can be added as the inverse of Larger.

Relations in the blocks language
Read pages 422-425.
It is much harder to axiomatize the 2 and 3 place relations in the blocks language. The book doesn't even bother. But we can do some things.

SameCol and SameRow are both equivalence relations. This means that the relation is reflexive, symmetric, and transitive.

1) Write out the sentences that would say that SameCol and SameRow are reflexive, symmetric, and transitive (three sentences for each predicate).

This is not a complete axiomatization for these predicates.

2) Give an example of a sentence that is always true of SameCol but not always of SameRow and

3) Give an example of a sentence that is always true of SameRow but not always of SameCol.

HINT: Use sentences that involves other predicates in the blocks language.

Part II: Axioms for Genealogy
Here some meaning postulates for genealogical relationships where P(x,y) is supposed to capture that x is a parent of y:

S: $\forall x \forall y(S(x,y) \leftrightarrow \exists z(P(z,x) \wedge P(z,y) \wedge x \neq y))$ *sibling*
G: $\forall x \forall y(G(x,y) \leftrightarrow \exists z(P(x,z) \wedge P(z,y)))$ *grandparent*
U: $\forall x \forall y(U(x,y) \leftrightarrow \exists z(P(z,y) \wedge S(z,x)))$ *uncle/aunt*
C: $\forall x \forall y(C(x,y) \leftrightarrow \exists z(P(z,y) \wedge U(z,x)))$ *first cousin*

You can prove a good number of things from these postulates. Prove that each of the following follows from S-C. To do this, start from no premises, but anytime you feel like it, you may write any of S-C and simply cite "S" or "G" or whatever is appropriate.

1) If Adam has a cousin, then one of his parents has a sibling.
$\exists x\, C(x,a) \rightarrow \exists x(P(x,a) \wedge \exists y\, S(y,x))$
2) First cousins share a grandparent. $\forall x \forall y(C(x,y) \rightarrow \exists z(G(z,x) \wedge G(z,y)))$

3) Using the above predicates, formalize the claim that for any two of Bettie's grandchildren, these two must be either siblings or cousins.

4) Prove claim #3

These postulates never allow you to prove any 'negative' claims (except that 'sibling' is irreflexive). For example, each of the following is consistent with S-C: Adam is his own uncle, Bob's uncle is the parent of Bob's grandfather, Christine has a sibling who is also her cousin and also her grandparent.

You might think that these should be logically ruled out. But each of those is consistent if you have the right kind of incestuous relationships. On the other hand, barring time travel, the meaning of parent does rule out the following: Angie is her own parent, her own grandparent (by blood not marriage), etc. We need 'P' axioms to take care of these.

Parenthood is notoriously difficult to axiomatize (impossible in my opinion). Without any axioms, you have the following extremely simple problem: P(a,a) might be true. To rule that out, we could add $\forall x\, \neg P(x,x)$. That is, nobody is their own parent. But then it is still consistent that you are the parent of your parent. We want to rule that out so we could add $\forall x \forall y(P(x,y) \rightarrow \neg P(y,x))$. But actually, you don't need both.

5) Show that $\forall x \forall y(P(x,y) \rightarrow \neg P(y,x))$ [lets call this A1] implies that no one can be their own parent $\forall x\, \neg P(x,x)$

6) Show that A1 does rule out being your own grandparent. That is, give a proof that $\forall x \forall y(P(x,y) \rightarrow \neg P(y,x))$ + G entail $\neg \exists x\, G(x,x)$.

7) This single axiom is not complete. Give a model of A1 interpreting P(x,y) as an arrow pointing from x to y which is consistent with someone being the parent of their own grandparent. Obviously to show this, you need to use the definition G. So give your

diagram with just the P arrows and name the objects and then also list who are the relevant objects and relationships that make someone the parent of their own grandparent.

8) We want to block this model as well. So lets just explicitly add the axiom that no one is a parent of their grandparent $\neg\exists x\exists y(P(x,y) \wedge G(y,x))$. Call this A2. First, give a proof that this is equivalent to adding $\forall x\forall y\forall z[(P(x,y) \wedge P(y,z)) \rightarrow \neg P(z,x))]$. To do this, show that G+A2 implies $\forall x\forall y\forall z[(P(x,y) \wedge P(y,z)) \rightarrow \neg P(z,x))]$ and also that G+$\forall x\forall y\forall z[(P(x,y) \wedge P(y,z)) \rightarrow \neg P(z,x))]$ entails $\neg\exists x\exists y(P(x,y) \wedge G(y,x))$.

9) But this is not enough either. Give a model of A1+A2 which shows that it is possible that you can be the grandparent of your own grandparent (give an interpretation as in #6)